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No. 2206: February 8, 1922

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The Texas Mathematics Teachers' Bulletin

Volume VII, Number 2

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The benefits of education and of useful knowledge, generally diffused through a community, are essential to the preservation of a free government.

Sam Houston

Cultivated mind is the guardian genius of democracy. . . . It is the only dictator that freemen acknowledge and the only security that freemen desire.

Mirabeau B. Lamar

University of Texas Bulletin
No. 2206: February 8, 1922

The Texas Mathematics Teachers' Bulletin

Volume VII, No. 2

Edited by

P. M. BATCHELDER
Instructor in Pure Mathematics,

and

A. E. COOPER
Instructor in Applied Mathematics

This Bulletin is open to the teachers of mathematics in Texas for the expression of their views. The editors assume no responsibility for statements of facts or opinions in articles not written by them.

PUBLISHED BY THE UNIVERSITY FOUR TIMES A MONTH AND ENTERED AS SECOND-CLASS MATTER AT THE POSTOFFICE AT AUSTIN, TEXAS, UNDER THE ACT OF AUGUST 24, 1912

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WHAT THE COLLEGE OR UNIVERSITY EXPECTS THE FRESHMAN TO KNOW IN THE WAY OF MATHEMATICS*

By H. J. Ettlinger

The expectations of the college teacher of mathematics with respect to the mathematical knowledge of freshmen just matriculating should not be very high. That does not signify that in a given freshman class there are no students who can work substantial original problems in algebra and geometry. The Brown Freshman Prize Examination at the University of Texas has established the fact that each year there come to the University young men and young women who fulfil the highest expectations in this respect, and this state of affairs is undoubtedly true in every institution in the State of Texas. The fact remains, however, that the first hour quiz two weeks after the opening of the session discloses that not only fifty per cent but the vast majority of our freshmen, because of lapse of time and memory, and other complications, exhibit few mathematical scars remaining as vestiges from the study of mathematics to which they have been exposed. Since the vaccination did not take the first time and in the absence of proof of immunity it has become quite necessary to give the treatment anew, beginning afresh from the first syllables and the first steps to be learned.

It is not, however, with the shortcomings of those entering college that this paper is concerned. Nor is it intended to discuss the content of the freshman course at college with respect to the knowledge which we teachers of mathematics desire our students to have, especially those who are planning to specialize in this field. Rather the purpose is to present a comprehensive view of what the needs of the college or university as a whole are. In Texas, I believe, every collegiate institution requires one year of

*Presented to the Texas Section, Mathematical Association of America, at Dallas, November 25, 1921, as the address of the retiring chairman.

mathematics toward the A. B. degree. What expectations do the various departments of the university have regarding the mathematical information which students completing the required course may have? This deals not alone with the content of the freshman course, but with all the preparatory work preliminary to this course. Not only is the college teacher of mathematics interested in this inquiry, but it reaches the high school teacher as well. Let it be distinctly understood, however, that it is not the purpose of this paper to urge the teaching of those topics which others consider essential to the exclusion of everything else, but merely to serve as a guide and help in the matter of emphasis.

Recently the National Committee on Mathematical Requirements sent out a questionnaire to certain specialists in various departments of study covering the following points: I. The personal experience of the individual relative to the value of specified topics in mathematics in his own life and work; II. Amount of mathematics necessary for advanced work in the respondent's field; III. Opinion as to the disciplinary and cultural value of mathematics in general; IV. Value of specific topics. The results of this inquiry are not at hand. They are to be included in the final inclusive report of the National Committee and a summary is to appear in the next number of the *Mathematics Teacher.*

In the present instance, the questions were confined to IV, following the plan of the National Committee, to find the judgment of the university as a whole as to the value of specified topics. The usual topics of the freshman year and some of the preparatory years were listed and judgment solicited on each with reference to the value of each topic, (1) as preparation for elementary courses, (2) as preparation for advanced courses. Weights were assigned as follows, E=essential, C=of considerable value, S=of some value and O=of no value. The answers represent the following departments: physics, chemistry, economics, botany, geology, education, zoology, psychology, philosophy, and business administration. Engineering was not included,

because the writer is convinced that the answer for at least ninety-five per cent of the topics would register an E=essential in both columns heavily underscored, for there every topic of the freshman mathematics course, almost without exception, has an immediate application in engineering. The same might have been suspected of physics and this was substantiated by the replies. Every one admits that mathematics is the *sine qua non* of these two sciences.

The topics listed include negative and imaginary numbers, their meaning and use, formulas and graphical representations, linear and quadratic functions and equations in one and two unknown numbers, arithmetic and numerical computation with reference to rational use of significant figures and approximations, logarithms, slide rule and numerical tables, theory of exponents and logarithms, arithmetic and geometric progressions, combinations and permutations, probability, statistical methods, solution of triangles by graphical and numerical methods, ruler and compass construction, mensuration, demonstrative geometry, plane trigonometry, and the topics of analytic geometry such as straight line, circle, conic sections, polar coordinates, and empirical curves, and elements of solid analytic geometry.

On one topic, viz., the understanding and use of simple formulas, the vote seems to be unanimous. It seems to be the general consensus of opinion that this topic is universally important. Every science after advancing beyond the descriptive stage attempts to formulate its results in terms of laws. This is but the mathematical expression of the quantitative relationship which exists between the data of experience, observation, or experiment. A striking example of this is the success of modern geneticists in expressing by means of a formula the relationship between the hereditary characters of an organism and the internal arrangement of the cell.

The labor of the National Committee on Mathematical Requirements has justified itself if it succeeds in leaving an indelible impress of the importance of making the formula or function idea the central unifying principle of the

high school course in algebra and also of the later work. Certainly the formula is typical of the mathematical mode of thought, and translation of the words of the English language into formulas and vice versa is fundamental. At the present time this topic is not sufficiently emphasized, as is witnessed by the fact that students are wofully weak on "word" problems. They could solve the equation, they say, but can not formulate the problem.

Computation and the solution of linear and quadratic equations are the natural concomitants of the work connected with formulas. Logarithms and the use of the slide rule are much in demand in every department of university work, especially recently in commercial work. For much of this work, a clear-cut conception of accuracy of approximation of the numbers involved both in the given data and the final result is wanted. Of considerable value is the verdict of all.

Everyone but the geologist stresses the importance of the graph as a mathematical instrument to be used to represent the dependence of one quantity on another, or to picture statistical data. As a means of solving problems the graph as a tool is found very useful. Ratio, proportion, and variation, closely allied to the formula both in interpretation and as a method of solution, are listed as of considerable value by some, by others as essential.

The geologist alone lists as important plane trigonometry, doubtlessly having in mind triangulation and problems connected with geodetic surveying. He lists as of no value whatsoever methods of computation, as if the geologist has no use for arithmetic whatever. This is an example of a peculiar habit of mind of some workers in science, who use mathematical methods and tools constantly without recognizing them as such. I have had the experience of hearing a professional educator tell me that correlation methods now so commonly used are not mathematics at all. It was not done in the spirit of taking away from mathematics a useful method whose applications are becoming more and more wide-spread; it was merely a case of blissful innocence, shall I call it, in the absence of a stronger word.

There is mixed feeling on such topics as the progressions, permutations and combinations, the binomial theorem, and probability, though it is to be pointed out that for advanced work each of these is labeled not less than of some value or of considerable value.

Intuitive geometry, such as familiarity with common geometric forms, direct and indirect measurements of lengths and angles, and constructions, is placed as having some or considerable value in all branches which require figures to be drawn. While not a part of the freshman course, this phase of high school plane geometry should be given more emphasis.

It was to be expected that the theoretical chapters, such as conception of a proof in demonstrative geometry, theory of exponents, theory of logarithms, special study of the straight line, circle, conics, etc., in analytical geometry, should not be scored very heavily and the records bear this out. That this means that these topics should be omitted or treated lightly by no means follows. There will be many who will ask whether the testimony here submitted is given by qualified witnesses. My own experience tells me that testimony of this kind is not always worth its face value. I have approached an engineering instructor with a view to cooperation and been met with the following statement, made without a trace of malice but with a good deal of sincerity: "Oh! the mathematics you teach my boys is merely perfunctory; if I want them to know any mathematics, I teach it to them myself." Merely said in the spirit that the only one qualified to teach mathematics to a student of engineering is an engineer. Again, I had a graduate student of chemistry having considerable difficulty with the beginning of calculus, who had already passed with honor a course in physical chemistry in which she had presumably solved differential equations. That men in other fields are competent judges of the amount of mathematics *they* use does not always follow.

What then should our course be? I do not believe in a fifty per cent proposition, in this case, but I believe that if we form a combination, made of twenty-five per cent of

information such as an inquiry of this sort gives and seventy-five per cent our own judgment, we shall be better able to do two things: (1) serve the devotees of mathematics, and (2) train good users of the science. Mathematics has been called both the queen and the handmaiden of the sciences. In closing permit me to quote an ancient sage: "If I am only for others, then who will be for me, but if I am only for myself then what am I?" In other words, if mathematics is only to serve other sciences then it loses its identity, but if it is only for itself then it loses a great part of its usefulness.

MATHEMATICS CLUBS FOR HIGH SCHOOLS

ARTHUR W. HARRIS

As time moves on we see more and more attention paid to the part of student activities in the training of future citizens. Not only have the number and size of clubs increased, but the variety has been greatly enlarged. Not long ago high school students had the choice of belonging to a debating society or remaining aloof from organized activity; now, many other clubs are offered in addition to literary societies. There are clubs promoted by the Young Men's Christian Association and the Young Women's Association among high school pupils. Glee clubs, bands, and orchestras are offered to the musically inclined. Athletic clubs foster the various sports, while woodcraft clubs call to the lover of nature's beauties. Then there are the organizations devoted to the study of mathematics or the various languages. It is concerning one of these newer clubs that I wish to write—the mathematics club.

Every high school of any considerable size should have a club devoted to the enlargement of the mathematics taught in our limited course of study. If mathematics is the center of scientific knowledge, surely it offers material around which we can build beneficial programs. Have you ever told your pupils that "this topic would be more interesting if we had the opportunity to carry it further"? Why not explain it to them through a mathematics club?

As results would probably be more interesting than theory, I shall mention a few facts concerning the club which we have in our high school. We call it *The Polygon,* because our work is many-sided. Our purpose is to study mathematics in such a way as to give the subject a higher place in the student's interest, thereby encouraging a deeper and lengthier study of it.

Most of our material has some direct connection with mathematics, though we avoid the classroom examples. Proofs of short cuts, such as the squaring of a number

ending in 5, often furnish a part of our program. A fallacy, such as the proof that a part of an angle is equal to the whole, is usually explained. Those present may be asked to arrange four 9's so that they will equal a hundred, or to solve some other puzzle. The lives of great mathematicians or other bits of history are subjects for interesting review. The recent methods proposed for the trisection of an angle were discussed in several meetings. The representation by graphs of statistics of current interest, such as immigration figures, prove very interesting as well as instructive. A message in code is offered for the members to decipher, followed by an explanation of the code used. One meeting each month is devoted to chess, a game which is an ally of mathematics in the development of the thinking powers. An occasional article on astronomy is enjoyable. A psychological test, sometimes in the form of a contest between boys and girls, is very entertaining. Lectures by business men regarding the application of mathematics in their occupations is a source of useful information. By varying these and other topics we secure interesting and instructive programs. This we believe is the secret of our success. A few social gatherings help to cement our membership into a congenial group.

To measure the results exactly is, of course, difficult; but it is certain that profit has accrued to the members of this club. They have learned to prepare for and take part in our programs; they have become acquainted with the rules of parliamentary law; they have had some valuable experience through committee work; and they have in most cases acquired a deeper interest in things mathematical.

My experience with *The Polygon* proves that the mathematics club deserves a high place among school activities.

[Mr. Harris is the mathematics teacher in the Bryan Street High School, Dallas.]

IS MATHEMATICS GOING THE WAY OF GREEK AND LATIN?

By Mary Beall McGee

Some time ago I heard an eminent professor of mathematics in one of our state schools make an astonishing remark to a group of college students. He had reference to the high schools and said, "The Greek has gone, Latin is going, much of the mathematics will follow." Present-day high schools offer no Greek in their course of study, many offer no Latin, and just a few now require Latin for graduation. This state of affairs has grown out of a broadening process to make elementary education fit the needs of the pupil. If mathematics fails to meet the pupil's needs it must necessarily go. What seems to be the present tendency? Two dominating facts are evident. First, mathematics has largely been placed among the elective courses; second, a large per cent of the students do not choose to elect mathematics. In view of these facts, I say a gradual process of automatic elimination is taking place. I am heartily in favor of offering mathematics as elective subjects, but in view of the unquestionable truth that many students are not electing the subject, I wish to discuss briefly, first, some of the more evident reasons for this tendency, and second, some things teachers of the subject can do, not only to retain mathematics as a vital part of the curriculum, but to make it a popular and valuable course.

For several years the disciplinary value of mathematics has been overestimated, while its importance in the actual workings of civilization has been badly neglected. Professor Earle Raymond Hedrick of the National Committee on Mathematical Requirements has recently said in a lecture to students of the University of Texas that a public untrained in mathematics is unable to appreciate things that affect him most and is incapable of efficient citizenship. In school circles the expression "training for citizenship" has become almost trite, yet the real value of mathematics con-

tinues to be neglected. With this misplaced emphasis mathematics must, inevitably, to a great extent fall out of our high school curriculum. A boy or girl can not be censured for not electing a subject when he can see nothing beyond a disciplinary value to be derived from his efforts. If the mathematics requirements for college entrance were broken down today I wonder where our algebra and geometry classes would be tomorrow. I'm forced to believe the mathematics classrooms would be vacant and the teachers would be in search of a job; I'm forced to believe we should wake up to the realization that mathematics is going the way of Greek and Latin.

The teachers of mathematics have been largely instrumental in the lack of interest manifested in the subject they teach. They are greatly to blame for the small per cent of pupils who choose to elect algebra and geometry. They, I claim, are responsible for the common notion that mathematics is difficult and only the so-called smart pupils can master it. I make these accusations because I believe the way mathematics has been taught is one of the greatest factors in the gradual elimination of the subject from our high schools. Too many teachers fail to realize the difference between telling and teaching. Most of us are guilty of saying to our pupils, "I have just told you so." Telling a thing can not be substituted for teaching it. Geometry seems hard to the beginner if he is suddenly confronted with a score of formal definitions all expressed in geometric language. Certainly he can not understand them, and naturally he thinks that what he can not understand is hard. At the very beginning the teacher has lost an opportunity to create in the pupil a lasting interest by failure to present new subject matter in a simple, common-sense manner. The pupils' vocabulary should be used as much as possible; it is oftentimes much more expressive than the language of the text. When the task of working out a proof is taken up it isn't sufficient to assign the work to the class to be mastered. First of all, it must not be overlooked that possibly the pupils may not know what a proof is. It must first be explained in the simplest way possible that a proof is a series

of facts leading from a definite hypothesis to a definite conclusion, each fact being defended by an axiom, definition, the hypothesis, or theorems already proved. Then the essential parts of a good proof may be presented and the work begun in an intelligent and understandable sort of way. Not only have teachers failed to present mathematics in a simple, understandable way, but they have overlooked the practical and economic side of the subject. It has sadly taken on the role of a classical subject with no mention made of the numerous possible applications. The best material for use in the advanced arithmetic and geometry classes may be had from every-day life, from circumstances that concern the pupil in a practical and material way. Too long teachers have failed to recognize the advantage to be gained by emphasizing the practical applications of the subject.

These conditions are a direct challenge to the high school teachers of mathematics. Much may be done to overcome them, even though too much emphasis has been unwisely placed on the disciplinary value of the subject, and in spite of the fact that teachers have failed to create a desire for the study of mathematics. Some teachers are popularizing the subject and in some schools the mathematics classes are full of enthusiastic pupils manifesting a keen interest in their work. This should be the aim of all teachers of mathematics. The first essential to successful teaching is that the teacher manifest such an interest that it will cause the pupils to show something of the same interest. An interested teacher has little trouble in devising methods for interesting the pupils. Some devices which have been successfully used will be discussed, since they are things that teachers of mathematics can do to keep the subject from going the way of Greek and Latin.

Kindergarten methods are not out of place in the geometry class. The pupils may be told to provide themselves with compass, ruler, construction paper, colored crayons, a pair of scissors, a bottle of glue and a ball of twine. Let them construct two triangles having two sides and the included angle of one equal respectively to two sides and the included angle of the other, cut these out, fit them together

and see for themselves that they will coincide in all their parts. By means of a piece of cardboard, a small stick cut from a chalk box, and a bit of twine, let the class beginning solid geometry construct a line perpendicular to two given lines at their point of intersection. They will be able to see, much more clearly than from the pictured figure, that the line is perpendicular to any other line in that plane which passes through the same point and is therefore perpendicular to the plane of the lines. The theorem is easily demonstrated with the constructed figure. The construction of figures for all propositions would, undoubtedly, be a waste of time, but for the untrained eye the method is invaluable. This brings up the question of notebooks. The more interesting constructions made in plane geometry, such as that showing that the square on the hypotenuse of a right triangle is equal to the sum of the squares on the other two sides, and that showing that if three circles are tangent externally at the points A, B, and C, and the chords AB and AC are produced to cut the circle BC at D and E, the line through D and E is a diameter, should be pasted in the notebooks. If the proposition is an original, the construction should accompany the proof. Notebook work, if properly directed, is an important factor in the teaching of geometry.

Every teacher should realize the value of a mathematics club and make an effort to organize one. Where it seems impractical to have a club, other methods may be used to the same advantage. Once or twice each term the class may be assigned a certain topic on which to prepare a paper. These papers should be read and graded by the teacher and the more interesting ones read aloud to the class. These papers should also be kept in the notebooks. The teacher will be guided in the assignment of topics by the reference material available. Unfortunately most schools have neglected the mathematics department in the selection of their library books. With a good encyclopedia, a good history of mathematics, and a few books such as Ball's *Mathematical Recreations and Essays*, and White's *A Scrap-Book of Elementary Mathematics*, a great deal of interesting material may be had. A few topics which have proved interesting

are: algebra of ancient Egypt, the meaning of the word geometry, calculating prodigies, magic squares, and the three famous problems of antiquity. If a club seems impossible it is time well used for the teacher to devote an occasional class period to the discussion of certain mathematical topics. Graphs such as are frequently found in *The Literary Digest* and other magazines may be profitably discussed, also the so-called mathematical recreations, and recent attempts to trisect an angle. Any topic which has a mathematical application will serve to create an interest, to link the pupil's mathematical knowledge to the outside world, and will be a diversion from the regular class work.

An excellent opportunity for the advanced arithmetic class lies in the management of a school store, the stock of goods comprising such articles as tablets, pencils, notebooks, ink, pens, etc. Practical experience is possible, and problems concerning buying and selling, profit and loss, the applications of percentage and many other phases of every-day arithmetic are made real. It is good if the class can borrow money to purchase their first stock and give a note for the amount, or in the larger places arrangements can be made with a local dealer who will likely offer a commission on goods sold. A set of books should be carefully kept by a bookkeeper appointed each month. His work should be checked by the pupils in the classroom, thus making the whole class responsible for the correctness of books. Without further discussion I offer the school shop as a suggestion to teachers of mathematics, because I know the plan presents unlimited possibilities for practical arithmetic and creates an interest otherwise unattainable.

Methods such as I have discussed will, I believe, help in a noticeable way to alleviate the causes for the present tendency of mathematics, and furthermore will cause high school pupils to realize that mathematics is not only interesting and worth while, but is essential to their elementary education.

Mary Beall McGee,
Bellville High School.

AN ACTUARY: HIS TRAINING, FUNCTIONS AND SERVICE*

By C. P. Rockwell

In the earliest days of life insurance, an actuary was simply a registrar or clerk. During the last few years, he has been thought of as one who was proficient in the practical application of the mathematics of life insurance (actuarial mathematics), including the theory of probability. However, his work is now acquiring a broader significance, and he might almost be classed as an "expert in insurance," it being necessary that he be well versed in higher mathematics, business law, accounting, and finance.

Only during the early part of his career will he be engaged in the compilation of tables, the determination of adequate premium rates, the consideration of equitable surrender charges and allowances, the calculation of reserve values, or the gathering of statistical data for present or future use. He must of necessity be immersed in figures, formulas, and mathematical problems, to some extent, and yet if he is to justify the boundless promise of his profession that he will "substitute facts for appearance and demonstrations for impressions," he must stretch out into still broader fields of labor and thought. As he gains in the practical application of his own theories, his intellectual horizon expands, and year by year he feels the growth of the faculty of accomplishing work of a higher character than he has ever before attempted.

The actuary must always be the man of figures—figures, ideas, and abstract theories must always remain with him, but the mathematical training in its strict application and interpretation is only one part of the equipment he needs to carry out successfully his life's work. He must possess as a talent, or else acquire by the slow process of patient

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*Address delivered before the Mathematical Association of America, at Dallas, Texas, November 25, 1921.

and painstaking study, the power of logically analyzing the varying conditions of the business as they come within the range of his observation.

Mathematical training, with patient study and practice, may make a perfect mathematician, but still fail to give us an actuary. To the born mathematical genius, or the person with acquired mathematical talent, must be added something that is absolutely essential to produce the actuary. That something includes a minute observation of the practical workings of life insurance, the ability to recognize and appreciate the weight and importance of business interests and business requirements—the wedding of theory to practice, of the ideal to the actual.

The actuary of today must be well versed in the scientific principles that underlie life insurance practice, but he can not win success unless he also acquires an every-day experience with its working details. Do not think for one moment that I seek to belittle the mathematical groundwork of the actuarial education. It is an absolute necessity. It is the very foundation upon which every other actuarial faculty is built, and if the foundation be weak, the superstructure must at some time fall to pieces. I merely wish to show that it is only the beginning—not the end. It is the foundation that supports the edifice raised upon its strength, but it is not the building itself. From these considerations, I reach the conclusion that not every mathematician is an actuary, although every actuary must of necessity be a mathematician.

Starting from the necessary foundation of a more or less intimate knowledge of the principles of mathematics, by the aid of which the student acquires an essential acquaintance with values and equivalents through which premiums, reserves, and other formulas become intelligible to him, he reaches successively the more or less complicated mathematical analyses that follow, until the various insurance functions are clear to his comprehension. The knowledge thus acquired is not of itself sufficient to constitute an expert in actuarial science, but it is the base or foundation upon which every future study and all future research can

be built together into one harmonious whole—foundation and superstructure being one and indivisible.

Having mastered the strictly mathematical groundwork, the mathematician who is aspiring to become an actuary will study the practical details of the life insurance business, critically analyzing all the circumstances that daily come under his observation. He must form the habit of independent thought and research, and come to definite and logical conclusions through his own thinking and on his own responsibility. The actuarial student must not advance one single step until all that he has left behind him is perfectly clear and intelligible. To learn that a certain formula constitutes the mathematical premium for a given form of insurance, without clearly understanding the logical and common sense operation and effect of it, is mere parrot learning. The actuary needs something more than to commit to memory a series of algebraic formulas or expressions; he needs clearly, intelligently, and comprehensively to recognize and understand every factor and every one of its legitimate functions.

The actuary is expected to calculate all premium rates, to vouchsafe their scientific accuracy and also their adequacy. He must determine the financial obligations involved in the various policy forms issued by his company, and he is directly responsible for their safety. It is thus necessary that he possess sound judgment, and also that he be able to gauge accurately the interests and intents of those whose points of view differ from his own.

The actuary collects and arranges statistics of the experience of his company and of the insurance world as a whole. He is usually expected to have a general knowledge of the laws of the various states pertaining to insurance, especially those giving the requirements for admission of life insurance companies to do business within their limits, and to see that his company and agents are licensed each year continuously, and also to see that the annual statements and other papers required by the various state insurance departments are properly executed.

The actuary will frequently assist the medical director in

passing upon risks, especially those involving questions of a moral nature, the hazard of occupation, or questionable personal habits and heredity. Furthermore, the medical director is often greatly indebted to the work of the actuary in collecting and analyzing statistics, for it is he who lays before the medical director the tabulated experience of his own and other companies. It is his studious research which enables him to point out the effect of certain diseases and hereditary tendencies upon the mortality experience. He is the one who enables the medical director to apply his noble science effectively, profitably, and to the best interests of life insurance.

Again, it is the actuary who realizes that all of the important clauses in a new policy form constitute causes from which must be determined the probable effects. This not only requires the exercising of his mathematical knowledge, but also of his judgment.

The actuary must determine accurately, and on strictly scientific principles, all policy values and the money value of all obligations connected therewith. He must decide the basis upon which any desired change in an existing policy contract can be made with perfect safety to the company. In fixing the cash value and other values of the equities of policyholders who give up their policies, he must exercise all possible skill to prevent his company from becoming the victim of adverse selection.

Another duty of the actuary lies in the individual distribution of the surplus, since when a determined amount of surplus is to be divided among a specific number of policyholders, it rests with him to see that such distribution is effected equitably and justly to all those who participate in it.

The actuary frequently shares with the other executive officers the responsibility of safely investing the funds of the company, protecting the interests of both the policyholders and the stockholders. He, being the officer directly responsible for the accuracy of the chief liability items in the financial statement, namely the policy reserve, is the one best fitted to see that the assets held to enable the payment

of these liabilities later to be effected are safely invested. A capable and conscientious actuary, well versed in every element of his duties, can not contemplate merely one side of the financial statement. He can not be content to define accurately the various reserve liability items, without thoroughly satisfying himself that each is fully covered by sound investments.

If there is any officer of an insurance company whose correspondence comes near touching upon every phase of human knowledge, it must be the actuary. The unusual questions and inquiries to which he must furnish a ready response almost surpass belief. The proper handling of these letters requires the constant use of business courtesy, and it frequently taxes his ingenuity and persuasive powers to turn a would-be enemy into a warm friend of the company.

In some companies, the actuary has the privilege of determining the rates of commission and other compensation to be paid to agents, also of supervising the writing of the canvassing literature of the company, but if these are among his duties, it is usually by dint of special privilege.

Such, in brief, are the functions of an actuary. Now to touch upon a number of the developments in life insurance which have taken place during the past few years, the chief credit for which belongs to those actuaries of this country who have been both progressive and broad-minded and who were able to see the trend of the times far in advance.

Until rather recently, life insurance benefits have been payable in a lump sum, which, upon the death of the insured, the company turned over to the beneficiary; it then canceled the policy, considered its obligation fulfilled, and the matter settled. Then came the installment policy, which distributed the policy-proceeds evenly over a given term of years. But this method of payment did not provide for the beneficiary who lived beyond the period in which the payments were made, and who thus reached old age without an income. Here, the actuary added an annuity clause, and the policy then provided for payments to be made annually over a stated number of years-certain, and thereafter during the

lifetime of the beneficiary. From an original offering of annual installments, he passed to the monthly installment, and as a result, these monthly payments made by the life insurance company to the beneficiary now take the place, in whole or in part, of the monthly payments contributed by the insured during his lifetime.

The next progressive step taken was the evolution of the waiver of premium clause, which provided that in case the insured became disabled prior to attaining the age 60, so that he no longer had an income, that all future premium payments were waived as long as his disability continued. But although this provision enabled the insured to keep his policy in full force and effect, yet it did not provide for his family, who had no income after he became disabled. To meet this situation, the disability provision was introduced, which provided for monthly payments to the insured so long as the insured remained disabled. The double indemnity provision was then introduced, in carrying out which the company agrees that should the death of the insured result from any one of a number of specified causes, that the amount payable shall be doubled. This clause is a purely arbitrary one, and in no way scientific from an actuarial viewpoint.

The life insurance field may roughly be divided into three sections—ordinary, industrial, and group insurance. Ordinary insurance, as written by the old line, legal reserve companies of the United States, falls into three main divisions: whole-life policies, which provide for the payment of the face amount only upon the death of the insured; term policies, which provide for the same payment only in case death occurs during a specified term of years, nothing being paid in case of survival; and endowment policies, which provide not only for the payment of the face amount upon the death of the insured during a fixed term of years, but also for the same payment at the end of the term, if the insured be then living.

Industrial insurance is a form of life insurance especially designed to meet the requirements of the wage-earning or

industrial population. It differs from other forms of life insurance in four ways:

1. The premiums are payable weekly, whereas in ordinary insurance they are payable annually, semi-annually, or quarterly.

2. The premiums are collected weekly from the homes of the insured, instead of being payable at the office of the company.

3. The amount of the insurance is "adjusted to the unit of premium," usually 5 cents, 10 cents, etc., up to 70 cents. Thus in industrial insurance, we speak of a 5-cent policy, if the weekly premium is 5 cents, whereas in ordinary life insurance, the unit, usually $1,000, is the amount of insurance.

4. Industrial insurance is issued on the lives of both children and adults, whereas ordinary life insurance is confined almost entirely to adult risks.

Group insurance is a form of life insurance which insures the lives of a group of employees under a blanket policy, at a reduced rate of premium, and generally without individual medical examination. It is a development of the past decade. In most cases the entire cost of the insurance is assumed by the employer, as evidence of his appreciation of efficiency and loyalty; and as an inducement of continuity of faithful service he may increase the amount of insurance semi-annually or annually during the term of employment. Customarily, the amount of insurance starts with $500 minimum after three months of service, and is increased annually $100 until a maximum of about $2,500 is reached. In some cases the amount of the insurance corresponds with the annual salary. The popularity of this class of insurance is shown by the fact that the group insurance annual premiums now in force in this country amount to over $18,000,000, and the group life insurance in force amounts to nearly $2,000,000,000. Since group insurance benefits can only be granted to groups of not less than fifty without medical examinations, what shall we say of the distinct gift bestowed on a certain percentage in each group that of a certainty could not pass any required test for individual

protection because of being unable to pass the physical examination, on account of impaired health or advanced years? Moreover, as each employee becomes older, he becomes less insurable individually. The owner of a group insurance certificate, even should he change his place of employment, is guaranteed continued protection on any regular plan of insurance without medical examination. This benefit often proves to be a very valuable asset to holders of group insurance certificates.

It is now possible, as a result of the co-operation of the actuary and the medical director, for life insurance protection on equitable terms to be obtained by those who do not come up to ordinary standards of health. The aim now of some of the largest companies is to make life insurance protection available to practically every one, and to exclude no one on grounds of health. One company, the New York Life, now has actual experience data on various classes of sub-standard lives covering a period of nearly twenty-five years, and this has enabled the chief actuary and the medical director to assign the impaired risks to various groups, the future mortality rate of each of which can be predetermined with very great accuracy. This company has prepared elaborate statistics, ratings, and tables, and worked out a schedule system of rating. The risk is classified under four heads —family history, personal history, physical condition, and habits. The rating of the risk is thus a scientific process. The fine feature of this method of passing on impaired lives is that the applicant's physical condition is only one of the four considerations. For this reason, the sizing up of the case for rating purposes should be the function of the actuary, rather than of the medical director.

It is held generally by actuaries that the hazards of occupation are not so serious as physical impairments. They feel that it is possible to measure much more accurately the extra hazard connected with a sub-standard occupation than to decide in advance just about how much more to charge the man whose physical condition is sufficiently below normal to place him in the sub-standard class. The handling of impaired risks is, like life insurance itself, a business

of averages. Impaired risks, like standard risks, must be classified in order to yield the company a profitable experience on the business. The writing of sub-standard business should be as exact and mathematically correct as the writing of standard business. No company knows how long any one man is going to live, but it can determine with more or less accuracy how long men with the same physical impairment are likely to live.

These are merely a few of the outstanding landmarks in which the services of the actuaries of the past and present have proven a blessing to the vast body of the insured through enabling them to enjoy the manifold advantages of life insurance. The research, the study, the patience, the ingenuity, and the invincible determination of the actuary have been equal to the demands of every emergency that has arisen, and these admirable faculties have enabled him successfully to weigh every varying requirement and responsibility that the phenomenal expansion of the business has forced him to meet.

I can pay no higher tribute to the cause of life insurance, which is the actuary's field of service, than to quote the definition of it, as given many years ago by Elizur Wright, the actuary and first insurance official of Massachusetts. He said, "Life insurance is the standing together, shoulder to shoulder, of hosts of manly men, to defend each other's homes from the enemy that shoots on the sly and in the dark. It is the realization of fraternity without the destruction of independence and individuality. It is charity without cant, which enriches the giver, and does not humiliate the receiver."

C. P. ROCKWELL,
Actuary, Texas Department of Insurance and Banking.

THE HIGH SCHOOL LIBRARY

We give the titles and publishers of a few books which we believe would be of considerable worth to any high school library. Only those dealing with some phase of practical mathematics have been listed, and care has been used to include none too advanced for the average high school student.

If a similar list dealing with the teaching of mathematics; history of mathematics; particular branches of pure mathematics—conics, solid geometry, determinants, etc.; or even mathematical recreation books, would be of interest, the editors of the *Bulletin* will be glad to compile such a list for any high school teacher desiring it.

Technical Algebra, H. W. Marsh, Wiley.

Technical Geometry, H. W. Marsh, Wiley.

Technical Trigonometry, H. W. Marsh, Wiley.

Plane Surveying, J. C. Tracy, Wiley.

Practical Mathematics, A. G. Cracknell, Longmans.

Practical Curve Tracing, R. H. Duncan, Longmans.

Principles of Insurance, W. F. Gephart, Macmillan.

Practical Shop Mechanics and Mathematics, J. F. Johnson, Wiley.

The Mathematics of Applied Electricity, E. H. Koch, Wiley.

Industrial Mathematics, H. W. Marsh, Wiley.

Mechanics Problems, Engineering News Company.

Calculus for Engineers, J. Perry, Arnold (London).

Mathematical Theory of Investments, E. B. Skinner, Ginn.

MATHEMATICS TO BE OFFERED IN THE UNIVERSITY DURING THE SUMMER SESSION OF 1922

First Term

Students who present trigonometry for entrance may not register for Pure Mathematics 1a; those who present solid geometry for entrance may not register for Pure Mathematics 1x. Pure Mathematics 1a, or entrance trigonometry, is prerequisite to 1c; and 1c is prerequisite to 1d.

1a. Plane Trigonometry.—Trigonometric functions of angles, identities, solution of triangles, inverse functions, circular measure, and logarithms. Four sections. Associate Professor Calhoun; Instructors Decherd, Batchelder, Wilder.

1b. Algebra.—Assumes a knowledge of the amount of algebra usually covered by a good high school, but will review some of the topics of the high school course. Especial stress will be laid on quadratic expressions and equations, the graph, logarithms, progressions, determinants, and the binomial theorem. Four sections. Instructors Decherd, Horton, Cleveland, Wilder.

1c. Introduction to Analytic Geometry.—Cartesian coördinates, plotting curves from their equations, the analytic geometry of the straight line and of the circle, polar coördinates. Prerequisite: Trigonometry. Associate Professor Rice.

1d. Analytic Geometry.—The parabola, the ellipse, the hyperbola, translation and rotation of axes, tangents, parametric equations, space geometry. Prerequisite: Pure Mathematics 1c. Instructor Cleveland.

1x. Solid Geometry.—Associate Professor Calhoun.

3a. A. Calculus.—Prerequisite: Three-thirds of Pure Mathematics 1, including 1c, or Applied Mathematics 1. Instructor Horton.

102. A. Fundamentals in Elementary Mathematics.—For students interested in the philosophy or the pedagogy of mathematics. The significance of the number-concept; some important proofs in algebra; brief survey of the foundations of geometry; geometrical construction; de Moivre's theorem; and trigonometry. Prerequisite: Two courses in mathematics. Associate Professor Dodd.

11a. i. A. Advanced Calculus.—Prerequisite: Pure Mathematics 3 or equivalent. Adjunct Professor Ettlinger.

11b. i. A. Advanced Calculus.—Prerequisite: Pure Mathematics 3 or equivalent. Adjunct Professor Ettlinger.

129. A. Theory of Least Squares.—Prerequisite: Pure Mathematics 3 or equivalent. Associate Professor Dodd.

13a. i. G. Difference Equations.—Prerequisite: Pure Mathematics 11 or equivalent. Instructor Batchelder.

13b. i. G. Differential Geometry.—Prerequisite: Pure Mathematics 11 or equivalent. Associate Professor RICE.

SECOND TERM

1a. Plane Trigonmetry.—As in the first term. Instructor COOPER.

1b. Algebra.—As in the first term. Instructor HOLMES.

1c. Introduction to Analytic Geometry.—As in the first term. Instructor COOPER.

1d. Analytic Geometry.—As in the first term. Instructor HOLMES.

3b. A. Calculus.—The work of this course is based upon a knowledge of the calculus given in Pure Mathematics 3a, and is a continuation of that course. Instructor JACOBS.

3c. A. Calculus.—Prerequisite: Pure Mathematics 3b. Instructor JACOBS.

11c. i. A. Advanced Calculus.—Prerequisite: Pure Mathematics 11ab or equivalent. Professor BENEDICT.

6a. i. A. Modern Algebra.—Prerequisite: Pure Mathematics 205 or equivalent. Lecturer MUSSELMAN.

14a. i. G. Infinite Series.—Prerequisite: Pure Mathematics 11 or equivalent. Professor BENEDICT.

13c. i. G. Advanced Mathematics.—Prerequisite: Two advanced courses in mathematics. Lecturer MUSSELMAN.